ECON/DARE Math Camp

Quiz – KEY

Fall 2013

*Show your work.

Problem 1

Find the first and second derivatives of the following functions. Using the results, identify the critical points and describe the function.

a) $f'(x) = 2x$, critical point: $x = 0$. $f'(x) < 0$ for $x < 0$ (decreasing over these values), and $f'(x) > 0$ for $x > 0$ (increasing over these values). $f''(x) = 2 > 0$. $f(x)$ is convex $\forall x \in D$.

b) $f'(x) = 3x^2 + 6x - 9 = 3(x+3)(x-1)$, critical points: $x = -3$ and $x = 1$. $f'(x) > 0$ (increasing) for $x < -3$, $f'(x) < 0$ (decreasing) for $-3 < x < 1$, and $f'(x) > 0$ (increasing) for $x > 1$. $f''(x) = 6x + 6$, inflection point: $x = (-1)$. $f''(x) < 0$ for $x < -1$, so $f(x)$ is concave for $x < -1$. $f''(x) > 0$ for $x > -1$, so $f(x)$ is convex for $x > -1$.

Problem 2

Find the first and second derivatives of the following functions.

a) $f'(x) = 4(\ln x)^3 \left(\frac{1}{x}\right) + 2e^{2x}$; $f''(x) = \frac{12(\ln x)^2 - 4(\ln x)^3}{x^2} + 4e^{2x}$

b) $f'(x) = -\frac{160}{x^3} - 4$; $f''(x) = \frac{480}{x^4}$

c) $f'(x) = -\frac{(x^2-1)(3x^2+3)}{(x^3+3x)^2} + \frac{2x}{x^3+3x}$;

$f''(x) = -\frac{12x^2}{(x^3+3x)^2} + \frac{2(x^2+1)(3x^2+3)^2}{(x^3+3x)^3} - \frac{2x(3x+3)}{(x^3+3x)^2} + \frac{2}{x^3+3x}$

Problem 3

a) $F(x) = -\frac{4}{x} + \frac{3}{7}x^{7/4} + C$

b) $q^* = 15$, and $p^* = 45$.

$$\begin{aligned} PS &= \int_0^{15} (45 - \frac{1}{5}q^2)dq \\ &= [45q - \frac{1}{15}q^3]\Big|_0^{15} \\ &= 45 * 15 - \frac{1}{15}(15)^3 \\ &= 450 \end{aligned}$$

Problem 4

$\varepsilon_d = -\frac{1}{2}$

Constant elasticity of demand: If price increases by 2%, then Q_d decreases by 1%.

Problem 5

a)

$x_1* = \frac{I}{2p_1}$, and $x_2* = \frac{I}{2p_2}$

b)

$\frac{\partial x_1^*}{\partial p_2} = 0$. The optimal x_1* is independent of p_2.

$\frac{\partial x_2^*}{\partial p_2} = -\frac{I}{2(p_2)^2} < 0$. Price and quantity demanded are inversely related.

Problem 6

$$dU = U_{x_1}dx_1 + U_{x_2}dx_2$$
$$dU = 1dx_1 + \frac{1}{x_2}dx_2$$

set $dU = 0$

$$MRS = \frac{dx_1}{dx_2} = -\frac{1}{x_2}$$

Problem 7

Solve the following:

a) $\begin{bmatrix} 5 & 8 & -2 \\ 1 & -3 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 6 & -2 \\ 10 & -1 \end{bmatrix} = \begin{bmatrix} 28 & -9 \\ 22 & -3 \end{bmatrix}$

b) Using $j = 2$

$$det \begin{bmatrix} 2 & 6 & 0 \\ -3 & 0 & 5 \\ 1 & 1 & 4 \end{bmatrix} = 6(-1)^{1+2}A_{12} + 0(-1)^{2+2}A_{22} + 1(-1)^{3+2}A_{32}$$
$$= 92$$